THEOREM OF PYTHAGORAS

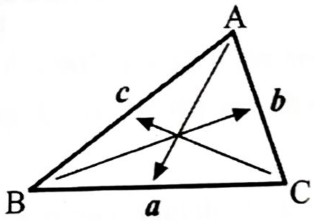
CHAPTER 13: MEASUREMENT

TOPIC: THE THEOREM OF PYTHAGORAS

In this chapter, we will revise the famous Theorem of Pythagoras that you studied in Grade 8. This theorem was first proved by the ancient Greek mathematician Pythagoras of Samos, who lived from about 569 BC to about 475 BC. The theorem was known to the ancient Babylonians 1000 years prior to Pythagoras. Ancient Indians (800 BC) and Chinese mathematicians (500 BC) were also aware of this theorem. This theorem is used extensively in so many areas of Mathematics such as Algebra, Trigonometry, Measurement, Analytical and Euclidean Geometry.

Let's briefly revise some important concepts from Grade 8 before exploring this theorem in more detail.

Naming the sides of triangles



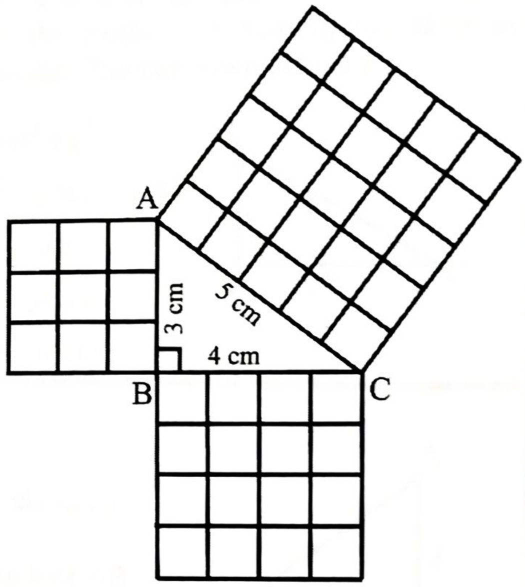
The sides of ΔABC can be named in terms of a, b and c.

In the diagram, the side opposite $\hat{A}$ is BC and we let $BC = a$.

The side opposite $\hat{B}$ is AC and we let $AC = b$.

The side opposite $\hat{C}$ is AB and we let $AB = c$.

How the Theorem of Pythagoras works.



Suppose that in ΔABC, $\hat{B} = 90°$, $AB = 3$ cm and $BC = 4$ cm.

On side AB create a square made up of 9 little squares each with an area of 1 cm^2.

On side BC create a square made up of 16 little squares.

It is now possible to create a square on the hypotenuse made up 25 little squares.

The area of the square on side AB
$= (3 \text{ cm})(3 \text{ cm}) = (3 \text{ cm})^2 = 9 \text{ cm}^2$

The area of the square on side BC
$= (4 \text{ cm})(4 \text{ cm}) = (4 \text{ cm})^2 = 16 \text{ cm}^2$

The area of the square on side AC
$= (5 \text{ cm})(5 \text{ cm}) = (5 \text{ cm})^2 = 25 \text{ cm}^2$

From the above it should be clear that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

If we now consider the sides of the triangle:

$AB^2 + BC^2 = (3 \text{ cm})^2 + (4 \text{ cm})^2 = 9 \text{ cm}^2 + 16 \text{ cm}^2 = 25 \text{ cm}^2$ and $AC^2 = (5 \text{ cm})^2 = 25 \text{ cm}^2$

$\therefore AC^2 = AB^2 + BC^2$

This relationship between the hypotenuse and the other two sides is referred to as the Theorem of Pythagoras. Let's state a rule that applies to all right-angled triangles.

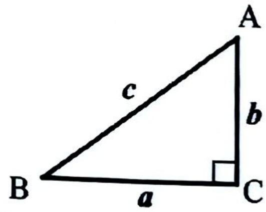
The Theorem of Pythagoras

In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

$AB^2 = BC^2 + AC^2$ or $c^2 = a^2 + b^2$

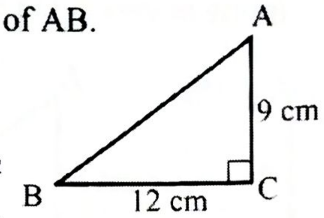
$BC^2 = AB^2 - AC^2$ or $a^2 = c^2 - b^2$

$AC^2 = AB^2 - BC^2$ or $b^2 = c^2 - a^2$



Example 1

Calculate the length of AB.



Solution

In ΔABC, $\hat{C} = 90°$

$\therefore AB^2 = BC^2 + AC^2$

$\therefore AB^2 = (12)^2 + (9)^2$

$\therefore AB^2 = 144 + 81$

$\therefore AB^2 = 225$

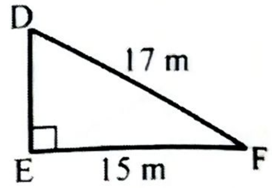
$\therefore AB = \sqrt{225}$

$\therefore AB = 15$ cm

> Note: $AB \neq -15$ cm since AB is a length which is always positive.

Example 2

Calculate the length of DE.



Solution

In ΔDEF, $\hat{E} = 90°$

$\therefore DE^2 = DF^2 - EF^2$

$\therefore DE^2 = (17)^2 - (15)^2$

$\therefore DE^2 = 289 - 225$

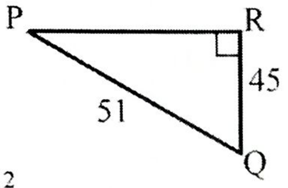
$\therefore DE^2 = 64$

$\therefore DE = \sqrt{64}$

$\therefore DE = 8$ m

Example 3

Calculate the length of PR.



Solution

In ΔPQR, $\hat{R} = 90°$

$\therefore PR^2 = PQ^2 - RQ^2$

$\therefore PR^2 = (51)^2 - (45)^2$

$\therefore PR^2 = 2\ 601 - 2\ 025$

$\therefore PR^2 = 576$

$\therefore PR = \sqrt{576}$

$\therefore PR = 24$ units

Example 4

In ΔABC, $\hat{B} = 90°$, $c = 18$ units and $a = 24$ units. Calculate the length of b.

Solution

First draw a rough sketch of the triangle using the method of naming the sides as discussed. The unknown side is b.

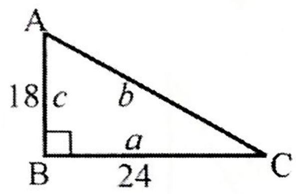
$b^2 = a^2 + c^2$

$\therefore b^2 = (24)^2 + (18)^2$

$\therefore b^2 = 900$

$\therefore b = \sqrt{900}$

$\therefore b = 30$ units

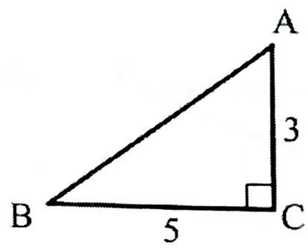


Example 5

In ΔABC, $\hat{C} = 90°$, $AC = 3$ and $BC = 5$.

(a) Calculate the length of AB without using a calculator. Leave your answer in surd form.

(b) Now use your calculator to calculate the length of AB. Round off to two decimal places.



Solution

(a) In ΔABC, $\hat{C} = 90°$

$\therefore AB^2 = BC^2 + AC^2$

$\therefore AB^2 = (5)^2 + (3)^2$

$\therefore AB^2 = 25 + 9$

$\therefore AB^2 = 34$

$\therefore AB = \sqrt{34}$ units

(b) Using a calculator:

$AB = 5{,}83$ units

EXERCISE 1

(a) Calculate the length of the unknown side in each of the following triangles. (The triangles are not drawn to scale).

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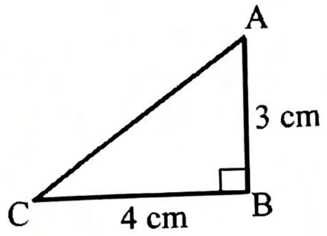

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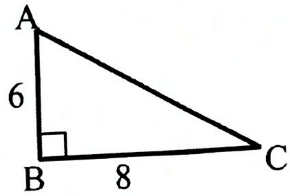

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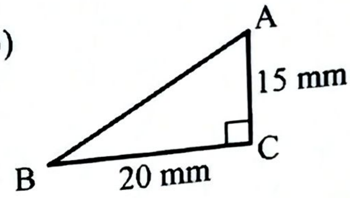

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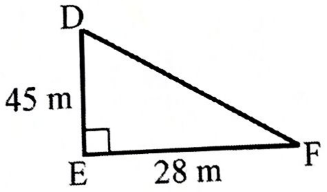

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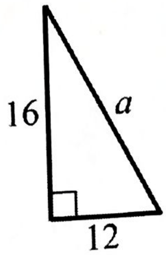

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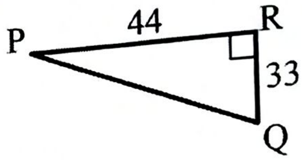

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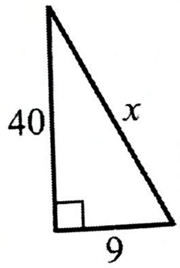

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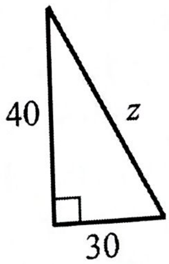

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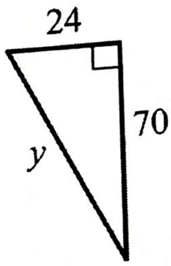

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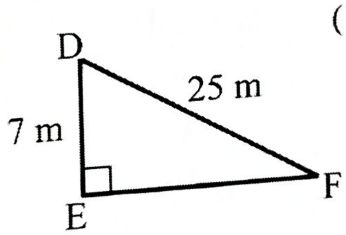

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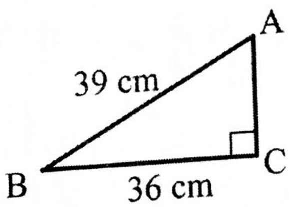

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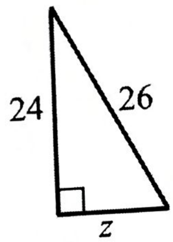

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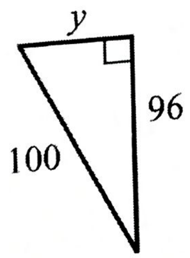

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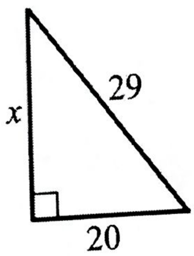

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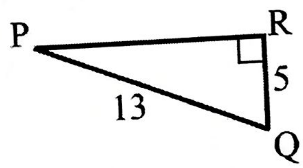

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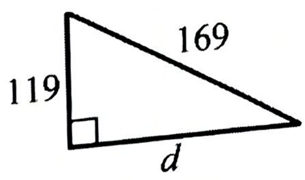

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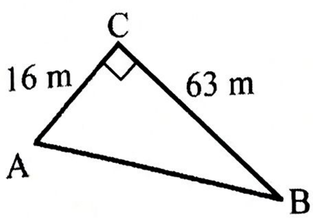

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